Sesión Especial 17

Junior GESTA (Geometría Simpléctica con Técnicas Algebraicas)

Organizadores

- Giovanni Bazzoni (Universidad Complutense de Madrid)
- Eduardo Fernández (Instituto de Ciencias Matemáticas & Universidad Complutense de Madrid)
- Cédric Oms (Universitat Politècnica de Catalunya)

Descripción

La Geometría Simpléctica es una de las ramas de la Geometría Diferencial que más se ha desarrollado en las últimas tres décadas. Este empuje se ha dado a raíz de ciertos problemas clásicos en Mecánica Hamiltoniana, en particular la conjetura de Arnold, que han sido resueltos con técnicas novedosas desarrolladas, entre otros, por Gromov y Floer.

Distintos enfoques son posibles en esta disciplina: algunos tienen un sabor más topológico y relacionan la Geometría Simpléctica con la Geometría de Contacto, especialmente en dimensión baja. Otros estudian las propiedades de compatibilidad entre una estructura simpléctica y otras estructuras geométricas en una variedad dada. Es la cooperación entre estos distintos puntos de vista que hace la Geometría Simpléctica tan exitosa.

En la sesión Junior GESTA pretendemos dar a jóvenes investigadores/as y estudiantes de doctorado, de ámbito nacional y europeo, la oportunidad de presentar los últimos resultados de su propia investigación en las distintas ramas de la Geometría Simpléctia, con el objetivo de fortalecer la red de jóvenes investigadores/as en dicho campo.

Programa

Lunes, 4 de febrero (mañana)

11:30 – 12:00	Luis Hernández Corbato (Universidad Politécnica de Madrid) *Tight neighborhoods of contact submanifolds*
12:00 – 12:30	Samuel Ranz Castañeda (Instituto de Ciencias Matemáticas) *On contact pencils*
12:30 – 13:00	Javier Martínez Aguinaga (Instituto de Ciencias Matemáticas & Universidad Complutense de Madrid) *Loops of Legendrians in the 3–sphere*
13:00 – 13:30	Ángel González Prieto (Universidad Politécnica de Madrid) *Hodge theory via Topological Quantum Field Theories*

LUNES, 4 de febrero (tarde)

17:00 – 17:30	Alfonso Tortorella (Katholieke Universiteit Leuven) *Contact dual pairs*
17:30 – 18:00	Nicoletta Tardini (Università degli Studi di Firenze) *Cohomologies of locally conformally symplectic manifolds and Hard-Lefschetz condition*
18:00 – 18:30	Antonio Otal (Centro Universitario de la Defensa, Zaragoza) *On Gauduchon connections with Kähler-like curvature*
18:30 – 19:00	Adela Latorre (Centro Universitario de la Defensa, Zaragoza) *Stability of pseudo-Kähler structures and related topics*

MARTES, 5 de febrero (mañana)

11:30 – 12:00	Lucía Martín Merchán (Universidad Complutense de Madrid) *Some examples of manifolds with* Spin(7) *structure*
12:00 – 12:30	Stefan Vasilev (Philipps-Universität Marburg) *Notes on the Wallach flag manifolds*
12:30 – 13:00	Roisin Braddell (Universitat Politècnica de Catalunya) *Group symmetries of cosymplectic and b-symplectic manifolds*
13:00 – 13:30	Robert Cardona (Universitat Politècnica de Catalunya) *A Moser trick for volume forms with singularities*

MARTES, 5 de febrero (tarde)

17:00 – 17:30	Michaël Orieux (Université Paris Dauphine) *Sufficient conditions for control-affine optimal time systems*
17:30 – 18:00	Rodrigo Schaefer (Universitat Politècnica de Catalunya) *Arnold diffusion for a Hamiltonian with 3 + 1/2 degrees of freedom*
18:00 – 18:30	Anastasia Matveeva (Universitat Politècnica de Catalunya) *Something about Stokes groupoids*
18:30 – 19:00	Juan Margalef Bentabol (Instituto de Ciencias Matemáticas) *Geometry, constraints, and boundaries. It takes three to tango*

Group symmetries of cosymplectic and b-symplectic manifolds

Roisin Braddell

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Abstract. Cosymplectic manifolds arise naturally in mathematical physics as time-flow of phase spaces. In certain cases the associated symplectic mapping torus can have monodromy. Cosymplectic mapping tori have also become of interest due to the connection to symplectic manifolds with singluarities, which are known as b-symplectic manifolds. Inspired by similar results from symplectic theory, we give a normal form result for cosymplectic and b-symplectic manifolds equipped with a group action in the neighbourhood of a group orbit.

A Moser trick for volume forms with singularities

Robert Cardona

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Abstract. Moser proved in 1965 that two different volume forms on a compact manifold are equivalent if and only if their cohomology classes coincide, using what has been called Moser's trick. In this talk I will present a generalizations of this proof for volume forms with singularities. When the volume form vanishes in a transversal way along a hypersurface, the needed hypothesis can be given in terms of relative cohomology. The 2-dimensional case is of special interest as the form can be seen as a symplectic structure with singularities. We will compare this to other classification schemes in (b)-symplectic geometry using the technique of deblogging.

Joint work with Eva Miranda.

Hodge theory via Topological Quantum Field Theories

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Abstract. Topological Quantum Field Theories (TQFTs) are powerful categorical tools that provide deep insight into the behaviour of topological invariants under gluing. In this talk, we will review some properties of TQFTs, emphasizing their lax counterparts and construction methods. As an application, we will construct a lax monoidal TQFT that computes the Hodge structure on the cohomology of representation varieties. For that, we will use a quantization procedure by means of Saito's mixed Hodge modules.
This TQFT gives an effective method of computation of E-polynomials of character varieties improving the prexisting methods. Furthermore, it offers a new framework in which mirror symmetry conjectures for E-polynomials may be addressed.

Joint work with M. Logares and V. Muñoz.

Tight neighborhoods of contact submanifolds

Luis Hernández Corbato

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Abstract. We prove that any small enough neighborhood of a closed contact submanifold is always tight under some assumptions on its normal bundle (hopefully none by the time of the talk). The non-existence of C^0–small positive loops of contactomorphisms in general overtwisted manifolds is shown as a corollary.

Joint work with Lucía Martín Merchán and Francisco Presas.

Stability of pseudo-Kähler structures and related topics

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Abstract. Let M be a $2n$-dimensional differentiable manifold. A pseudo-Kähler structure on M is a pair (J, ω), where J is a complex structure and ω is a symplectic form, satisfying the compatibility condition

$$\omega(J\cdot, J\cdot) = \omega(\cdot, \cdot)\,.$$

When $g(\cdot, \cdot) = \omega(\cdot, J\cdot)$ is a positive-definite metric, the manifold is Kähler and a well-known result by Kodaira-Spencer ensures stability along sufficiently small deformations of the complex structure. For pseudo-Kähler manifolds, if one considers another complex structure $\tilde{J}$ on M "sufficiently close" to J, then there might no longer exist a symplectic form on M compatible with $\tilde{J}$. For this reason, one would like to find conditions ensuring the stability of pseudo-Kähler structures. In this talk, we will address this problem and study certain conditions that guarantee such property.

Geometry, constraints, and boundaries. It takes three to tango

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Abstract. Constraints appear quite naturally in many interesting physical systems such as EM, gravity, or parametrized systems. If we are interested in their quantization, it seems necessary to have a precise Hamiltonian formulation. The Dirac's algorithm addresses this problem quite nicely when no boundaries are present, but... boundaries are also very natural! Through some examples I will show the limits of the Dirac's algorithm in the presence of boundaries and introduce an alternative algorithm, called GNH, to tackle these systems.

Loops of Legendrians in the 3–sphere

Javier Martínez Aguinaga

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Abstract. In this talk we will discuss some recent work regarding flexibility/rigidity in the space of legendrian submanifolds of the contact 3–sphere.

Joint work with Eduardo Fernández (UCM-ICMAT) and Francisco Presas (ICMAT).

Some examples of manifolds with Spin(7) structure

Lucía Martín Merchán

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Abstract. The group Spin(7) appears on Berger's list as exceptional holonomy group of 8-dimensional Riemannian manifolds which are simply connected, irreducible and non-symmetric. A homotopical obstruction for Spin(7) holonomy is the presence of a Spin(7) structure, that is, a 4-form Ω determining a triple cross product on the manifold.
The 5-form $d\Omega$ measures how far is the manifold from having Spin(7) holonomy. For this reason, Spin(7) structures are classified according to differential equations for Ω into four classes. In this talk, we will focus on the construction of examples of some of those types.

Something about Stokes groupoids

Anastasia Matveeva

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Abstract. I will explain the notion of Stokes groupoids. I will show how they can be associated with flat connections in locally trivial bundles over Riemann surfaces with boundary in the sense of Riemann-Hilbert correspondence and, if there is time left, I will say something about Poisson structures on both sides of the correspondence.

Sufficient conditions for control-affine optimal time systems

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Abstract. When one's interested in the minimum time control of mechanical systems, and more generally of dynamics that are affine in the control, necessary conditions give the optimal trajectory as the projection of the integral curves of an Hamiltonian system defined on the cotangent bundle of the phase space. Those are called extremal, and they lack regularity: the Hamiltonian is not smooth and has codimension 2 singularities. In this talk we will prove sufficient conditions for optimality of these singular extremals. Our methods uses technics from symplectic geometry, and consists in choosing a good Lagrangian submanifold on which the canonical projection of the extremal flow is invertible. Then one can compare final time of neighboring trajectories by lifting them to the cotangent bundle and evaluate the Poincaré-Cartan form along their lifts. The main difficulties is the definition of these objects without the required regularity, and a extended study of the extremal flow is necessary.

On Gauduchon connections with Kähler-like curvature

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Abstract. We study Hermitian metrics with a Gauduchon connection being "Kähler-like", namely, satisfying the same symmetries for curvature as the Levi-Civita and Chern connections. In particular, we investigate 6-dimensional solvmanifolds with invariant complex structures with trivial canonical bundle and with invariant Hermitian metrics. The results for this case give evidence for two conjectures that are expected to hold in more generality: first, if the Bismut connection is Kähler-like, then the metric is pluriclosed; second, if another Gauduchon connection, different from Chern or Bismut, is Kähler-like, then the metric is Kähler.

On contact pencils

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Abstract. We construct examples of strong foliated 5–folds that are C^0 limits of sequences of contact structures. The result is based on the use of generalised pencil decompositions for almost contact structures.

Arnold diffusion for a Hamiltonian with 3 + 1/2 degrees of freedom

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Abstract. In the present paper we study the geometrical mechanism of diffusion in an *a priori* unstable Hamiltonian system with 3 + 1/2 degrees of freedom

$$H_\varepsilon(p,q,I_1,I_2,\varphi_1,\varphi_2,s) = \pm\left(\frac{p^2}{2}+\cos q-1\right)+h(I_1,I_2)+\varepsilon f(q)\,g(\varphi_1,\varphi_2,s), \qquad (1)$$

where $f(q)=\cos q$, $h(I_1,I_2)=\Omega_1 I_1^2/2+\Omega_2 I_2^2/2$, $(\varphi_1,\varphi_2)\in\mathbb{T}^2$ and

$$g(\varphi_1,\varphi_2,s)=a_1\cos\varphi_1+a_2\cos\varphi_2+a_3\cos s. \qquad (2)$$

Combining iterates of the *inner* and the *outer* dynamics associated to a 5D-*Normally Hyperbolic Invariant Manifold* (NHIM) to build a diffusing *pseudo-orbit* and applying Shadowing results we prove the existence of a diffusing orbit of the system. More precisely, we are able to prove the following theorem on global instability:

Theorem. *Consider the Hamiltonian* (1)+(2). *Assume* $a_1a_2a_3\neq 0$ *and* $|a_1/a_3|+|a_2/a_3|<0.625$. *Then, for every* $\delta<1$ *and* $R>0$ *there exists* $\varepsilon_0>0$ *such that for every* $0<|\varepsilon|<\varepsilon_0$, *given* $|I_\pm|\leq R$, *there exists an orbit* $\tilde{x}(t)$ *and* $T>0$, *such that*

$$|I(\tilde{x}(0))-I_-|\leq\delta \quad \textit{and} \quad |I(\tilde{x}(T))-I_+|\leq\delta.$$

Cohomologies of locally conformally symplectic manifolds and Hard-Lefschetz condition

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Abstract. The Bott-Chern and the Aeppli cohomology groups provide useful global tools in studying non-Kähler geometry. We will introduce them on locally conformally symplectic manifolds discussing also their properties and applications. In particular, they are a generalization of the Bott-Chern and Aeppli cohomologies studied by L.-S. Tseng and S.-T. Yau on symplectic manifolds. We will also discuss the notion of Hard-Lefschetz condition in relation with the $d\delta$-lemma.

Joint work with Daniele Angella and Alexandra Otiman.

Contact dual pairs

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Abstract. In this work we investigate the notions of duality and dual pairs in Jacobi geometry. We introduce a contact dual pair as a pair of Jacobi bundle maps defined on the same (generically non-coorientable) contact manifold and satisfying a certain orthogonality condition. The standard example is formed by the source and the target maps of a contact groupoid. Among various properties, we investigate the relation existing between symplectic dual pairs and contact dual pairs via symplectization/Poissonization. Our main result is the proof of the characteristic leaf correspondence theorem for contact dual pairs. Indeed there is a one-to-one correspondence between the characteristic (symplectic or contact) leaves of the two Jacobi manifolds forming the legs of a contact dual pair with connected fibers. Finally we apply these results to the context of reduction theory. Indeed we prove that any free and proper contact groupoid action gives rise to the contact dual pair formed by the associated moment map and the projection on the orbit space. Consequently the characteristic leaf correspondence yields a new insight into the contact reduction method first introduced by Zambon and Zhu.

Joint work with A. Blaga, M.A. Salazar and C. Vizman.

Notes on the Wallach flag manifolds

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Abstract. In this ongoing work we investigate the Wallach flag manifolds $W^6 = U(3)/U(1)^3$, $W^{12} = Sp(3)/Sp(1)^3$, and $W^{24} = F_4/\mathrm{Spin}(8)$ from a G-structure point of view. These spaces are known for their appearance as exceptional examples of homogeneous spaces admitting metrics with strictly positive scalar curvature and also as examples of isoparametric hypersurfaces in spheres, and have been investigated extensively as such. The previous investigations link them closely to algebraic and geometric objects such as Jordan algebras, division algebras, cohomogeneity one orbits in spheres, and isoparametric hypersurfaces. Yet, from a G-structure point of view, we only have a good feeling of how W^6 looks. Extensive studies of W^{12} and W^{24} are lacking, and this is precisely what we are aiming at in our work. We use the existing unified algebraic description of the three spaces to seek after special geometric structures on them which can possibly generalize to other manifolds.
